PROCESS CAPABILITY ANALYSIS

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All figures are taken from(unless otherwise mentioned)
Introduction to Statistical process Control
Douglas. C Montgomery
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Introduction

- Statistical techniques can be helpful throughout the product cycle, including development activities prior to manufacturing, in quantifying process variability, in analyzing this variability relative to product requirements or specifications, and in assisting development and manufacturing in eliminating or greatly reducing this variability.
  - This general activity is called process capability analysis
- Process capability refers to the uniformity of the process.
- There are two ways to think of this variability:
  - 1. The natural or inherent variability in a critical-to-quality characteristic at a specified time; that is, “instantaneous” variability
  - 2. The variability in a critical-to-quality characteristic over time
- We define process capability analysis as a formal study to estimate process capability

Major uses of Process capability Analysis

- 1. Predicting how well the process will hold the tolerances
- 2. Assisting product developers/designers in selecting or modifying a process
- 3. Assisting in establishing an interval between sampling for process monitoring
- 4. Specifying performance requirements for new equipment
- 5. Selecting between competing suppliers and other aspects of supply chain management
- 6. Planning the sequence of production processes when there is an interactive effect of processes on tolerances
- 7. Reducing the variability in a process

Using Histogram – An Example

Figure 8.2 presents a histogram of the bursting strength of 100 glass containers. The data are shown in Table 8.1. What is the capability of the process?

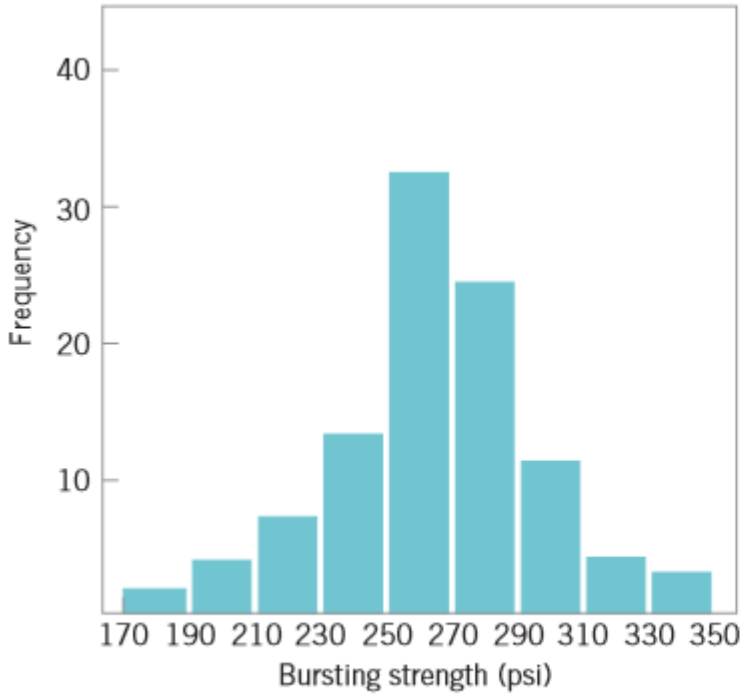


FIGURE 8.2 Histogram for the bursting-strength data.

TABLE 8.1

Bursting Strengths for 100 Glass Containers

265	197	346	280	265	200	221	265	261	278
205	286	317	242	254	235	176	262	248	250
263	274	242	260	281	246	248	271	260	265
307	243	258	321	294	328	263	245	274	270
220	231	276	228	223	296	231	301	337	298
268	267	300	250	260	276	334	280	250	257
260	281	208	299	308	264	280	274	278	210
234	265	187	258	235	269	265	253	254	280
299	214	264	267	283	235	272	287	274	269
215	318	271	293	277	290	283	258	275	251

Solution

Analysis of the 100 observations gives

$$\bar{x} = 264.06 \qquad s = 32.02$$

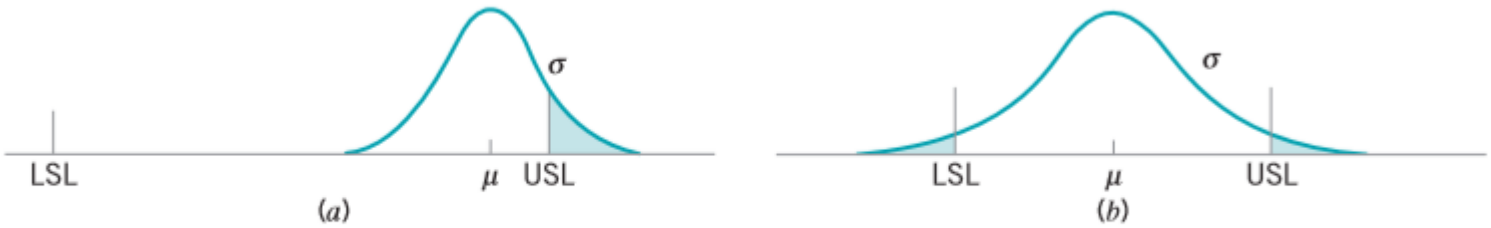
Consequently, the process capability would be estimated as

$$\bar{x} \pm 3s$$

or

$$264.06 \pm 3(32.02) \simeq 264 \pm 96 \text{ psi}$$

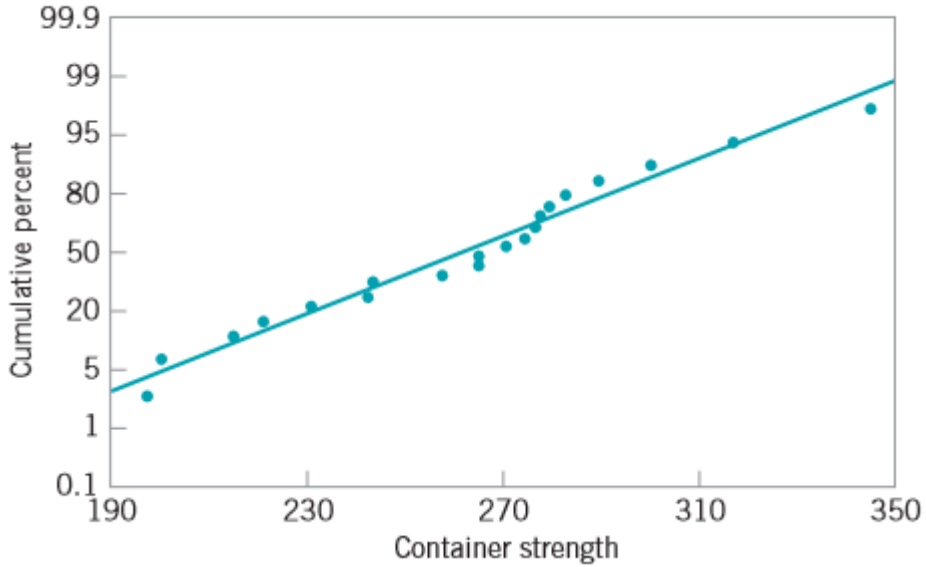
Furthermore, the shape of the histogram implies that the distribution of bursting strength is approximately normal. Thus, we can estimate that approximately 99.73% of the bottles manufactured by this process will burst between 168 and 360 psi. Note that we can estimate process capability *independent of the specifications on bursting strength.*



■ FIGURE 8.3 Some reasons for poor process capability. (*a*) Poor process centering. (*b*) Excess process variability.

Probability Plotting

- 20 observations on glass container bursting strength: 197, 200, 215, 221, 231, 242, 245, 258, 265, 265, 271, 275, 277, 278, 280, 283, 290, 301, 318, and 346



- Fig 8.4

Interpretation

- Mean of the normal distribution is the fiftieth percentile, which we may estimate from Fig. 8.4 as approximately 265 psi, and the standard deviation of the distribution is the slope of the straight line.
- It is convenient to estimate the standard deviation as the difference between the eighty-fourth and the fiftieth percentiles. For the strength data shown above and using Fig. 8.4, we find that $\hat{\sigma} = 84\text{th percentile} - 50\text{th percentile} = 298 - 265 \text{ psi} = 33 \text{ psi}$
- Note that and are not far from the sample average and standard deviation s = 32.02.
- Care should be exercised in using probability plots. If the data do not come from the assumed distribution, inferences about process capability drawn from the plot may be seriously in error

Process Capability Ratios

- Recall

$$C_p = \frac{\text{USL} - \text{LSL}}{6\sigma} \qquad (8.4)$$

In a practical application, the process standard deviation σ is almost always unknown and must be replaced by an estimate σ. To estimate σ we typically use either the *sample standard deviation* s or $\bar{R}/d_2$ (when variables control charts are used in the capability study). This results in an estimate of C_p—say,

$$\hat{C}_p = \frac{\text{USL} - \text{LSL}}{6\hat{\sigma}} \qquad (8.5)$$

The PCR C_p in equation (8.4) has a useful practical interpretation—namely,

$$P = \left(\frac{1}{C_p}\right)100 \qquad (8.6)$$

Process Capability Ratios

- Equations (8.4) and (8.5) assume that the process has both upper and lower specification limits. For one-sided specifications, one-sided process-capability ratios are used. One sided PCRs are defined as follows:

$$C_{pu} = \frac{\text{USL} - \mu}{3\sigma} \quad \text{(upper specification only)} \qquad (8.7)$$

$$C_{pl} = \frac{\mu - \text{LSL}}{3\sigma} \quad \text{(lower specification only)} \qquad (8.8)$$

An example

Construct a one-sided process-capability ratio for the container bursting-strength data in Example 8.1. Suppose that the lower specification limit on bursting strength is 200 psi.

We will use $\bar{x} = 264$ and $s = 32$ as estimates of μ and σ, respectively, and the resulting estimate of the one-sided lower process-capability ratio is

$$\hat{C}_{pl} = \frac{\hat{\mu} - \text{LSL}}{3\hat{\sigma}} = \frac{264 - 200}{3(32)} = 0.67$$

The fraction of defective containers produced by this process is estimated by finding the area to the left of $Z = (\text{LSL} - \mu)/\sigma = (200 - 264)/32 = -2$ under the standard normal distribution. The estimated fallout is about 2.28% defective, or about 22,800 nonconforming containers per million. Note that if the normal distribution were an inappropriate model for strength, then this last calculation would have to be performed using the appropriate probability distribution.

Values of Process Capability Ratio

Values of the Process Capability Ratio (C_p) and Associated Process Fallout for a Normally Distributed Process (in Defective ppm) That Is in Statistical Control

	Process Fallout (in defective ppm)	
PCR	**One-Sided Specifications**	**Two-Sided Specifications**
0.25	226,628	453,255
0.50	66,807	133,614
0.60	35,931	71,861
0.70	17,865	35,729
0.80	8,198	16,395
0.90	3,467	6,934
1.00	1,350	2,700
1.10	484	967
1.20	159	318
1.30	48	96
1.40	14	27
1.50	4	7
1.60	1	2
1.70	0.17	0.34
1.80	0.03	0.06
2.00	0.0009	0.0018

- Several values of the PCR Cp along with the associated values of process fallout, expressed in defective parts or nonconforming units of product per million (ppm)

Assumptions

- 1. The quality characteristic has a normal distribution.
- 2. The process is in statistical control.
- 3. In the case of two-sided specifications, the process mean is centered between the lower and upper specification limits.
- These assumptions are absolutely critical to the accuracy and validity of the reported numbers, and if they are not valid, then the reported quantities may be seriously in error.
- Stability or statistical control of the process is also essential to the correct interpretation of any PCR
- What we actually observe in practice is an estimate of the PCR. This estimate is subject to error in estimation, since it depends on sample statistics

Minimum Values for PCR

TABLE 8.3

Recommended Minimum Values of the Process Capability Ratio

	Two-Sided Specifications	One-Sided Specifications
Existing processes	1.33	1.25
New processes	1.50	1.45
Safety, strength, or critical parameter, existing process	1.50	1.45
Safety, strength, or critical parameter, new process	1.67	1.60

Process Capability Analysis using control chart

- Histograms, probability plots, and process capability ratios summarize the performance of the process. They do not necessarily display the potential capability of the process because they do not address the issue of statistical control
- The control chart should be regarded as the primary technique of process capability analysis.
- The x and R charts should be used whenever possible, because of the greater power and better information they provide relative to attributes charts.
- However, both p charts and c (or u) charts are useful in analyzing process capability.

An example

TABLE 8.5

Glass Container Strength Data (psi)

Sample	Data					$\bar{x}$	R
1	265	205	263	307	220	252.0	102
2	268	260	234	299	215	255.2	84
3	197	286	274	243	231	246.2	89
4	267	281	265	214	318	269.0	104
5	346	317	242	258	276	287.8	104
6	300	208	187	264	271	246.0	113
7	280	242	260	321	228	266.2	93
8	250	299	258	267	293	273.4	49
9	265	254	281	294	223	263.4	71
10	260	308	235	283	277	272.6	73
11	200	235	246	328	296	261.0	128
12	276	264	269	235	290	266.8	55
13	221	176	248	263	231	227.8	87
14	334	280	265	272	283	286.8	69
15	265	262	271	245	301	268.8	56
16	280	274	253	287	258	270.4	34
17	261	248	260	274	337	276.0	89
18	250	278	254	274	275	266.2	28
19	278	250	265	270	298	272.2	48
20	257	210	280	269	251	253.4	70
						$\bar{\bar{x}} = 264.06$	$\bar{R} = 77.3$

CALCULATION

$\bar{x}$ Chart

$$\text{Center line} = \bar{\bar{x}} = 264.06$$

$$\text{UCL} = \bar{\bar{x}} + A_2\bar{R} = 264.06 + (0.577)(77.3) = 308.66$$

$$\text{LCL} = \bar{\bar{x}} - A_2\bar{R} = 264.06 - (0.577)(77.3) = 219.46$$

R Chart

$$\text{Center line} = \bar{R} = 77.3$$

$$\text{UCL} = D_4\bar{R} = (2.115)(77.3) = 163.49$$

$$\text{LCL} = D_3\bar{R} = (0)(77.3) = 0$$

Figure 8.12 presents the $\bar{x}$ and R charts for the 20 samples in Table 8.5. Both charts exhibit statistical control. The process parameters may be estimated from the control chart as

$$\hat{\mu} = \bar{\bar{x}} = 264.06$$

$$\hat{\sigma} = \frac{\bar{R}}{d_2} = \frac{77.3}{2.326} = 33.23$$

Control Charts

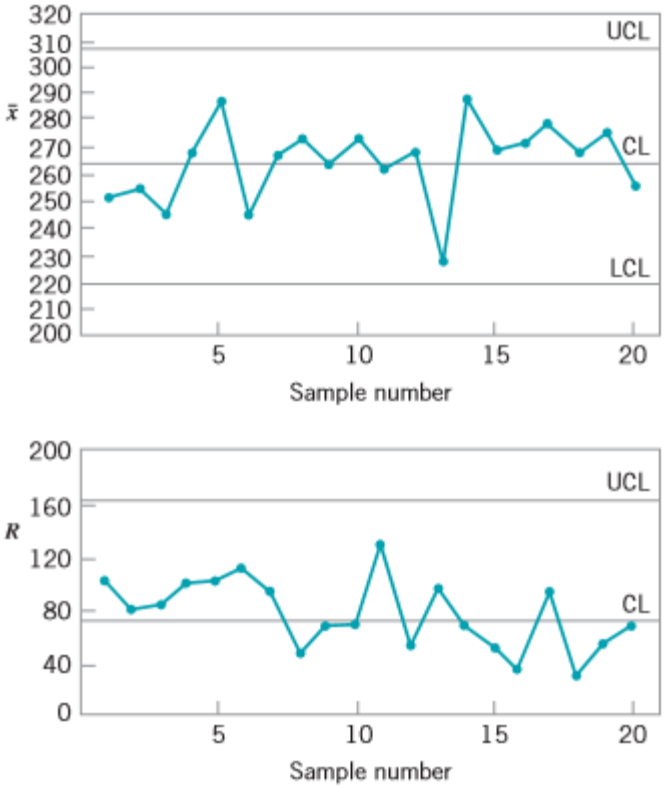


FIGURE 8.12 $\bar{x}$ and R charts for the bottle-strength data.

Thus, the one-sided lower process capability ratio is estimated by

$$\hat{C}_{pl} = \frac{\mu - \text{LSL}}{3\hat{\sigma}} = \frac{264.06 - 200}{3(33.23)} = 0.64$$

Conclusions

- This example illustrates a process that is in control but operating at an unacceptable level.
- There is no evidence to indicate that the production of nonconforming units is operator-controllable. Engineering and/or management intervention will be required either to improve the process or to change the requirements if the quality problems with the bottles are to be solved.
  - The objective of these interventions is to increase the process capability ratio to at least a minimum acceptable level.
- The control chart can be used as a monitoring device or logbook to show the effect of changes in the process on process performance. Sometimes the process capability analysis indicates an out-of-control process.
- It is unsafe to estimate process capability in such cases. The process must be stable in order to produce a reliable estimate of process capability.

Process Capability Using Designed Experiments

- A designed experiment is a systematic approach to varying the input controllable variables in the process and analyzing the effects of these process variables on the output.
- Designed experiments are also useful in discovering which set of process variables is influential on the output, and at what levels these variables should be held to optimize process performance.
- One of the major uses of designed experiments is in isolating and estimating the sources of variability in a process. For example, consider a machine that fills bottles with a soft-drink beverage. Each machine has a large number of filling heads that must be independently adjusted.
- The quality characteristic measured is the syrup content (in degrees brix) of the finished product. There can be variation in the observed brix (σ^2_B) because of machine variability (σ^2_M), head variability (σ^2_H), and analytical test variability (σ^2_A). The variability in the obs[illegible]lue is

$$\sigma_B^2 = \sigma_M^2 + \sigma_H^2 + \sigma_A^2$$

Explanation

- An experiment can be designed, involving sampling from several machines and several heads on each machine, and making several analyses on each bottle, which would allow estimation of the variances.

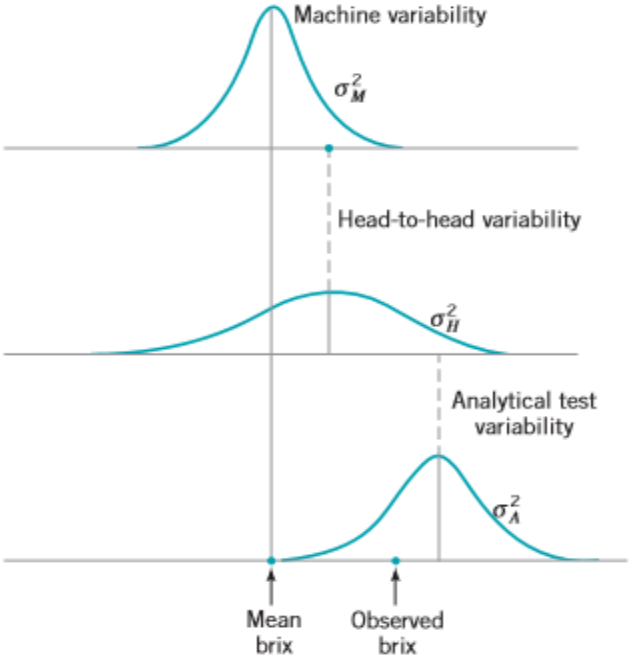


FIGURE 8.13 Sources of variability in the bottling line example.

Suppose that the results appear as in Fig. 8.13. Since a substantial portion of the total variability in observed brix is due to variability between heads, this indicates that the process can perhaps best be improved by reducing the head-to-head variability. This could be done by more careful setup or by more careful control of the operation of the machine.

Process Capability Analysis Using Attribute Data

- When dealing with nonconformities or defects, a defects per unit (DPU) statistic is often used as a measure of capability, where

$$\text{DPU} = \frac{\text{Total number of defects}}{\text{Total number of units}}$$

- Here the unit is something that is delivered to a customer and can be evaluated or judged as to its suitability. Some examples include: 1. An invoice 2. A shipment 3. A customer order 4. An enquiry or call.
- The defects or nonconformities are anything that does not meet the customer requirements, such as: 1. An error on an invoice 2. An incorrect or incomplete shipment 3. An incorrect or incomplete customer order 4. A call that is not satisfactorily completed

DPMO

- The DPU measure does not directly take the complexity of the unit into account. A widely used way to do this is the defect per million opportunities (DPMO) measure

$$\text{DPMO} = \frac{\text{Total number of defects}}{\text{Number of units} \times \text{Number of opportunities}}$$

- Opportunities are the number of potential chances within a unit for a defect to occur. For example, on a purchase order, the number of opportunities would be the number of fields in which information is recorded times two, because each field can either be filled out incorrectly or blank (information is missing).
- It is important to be consistent about how opportunities are defined, as a process may be artificially improved simply by increasing the number of opportunities over time.

Gauge Capability

- Generally, in any activity involving measurements, some of the observed variability will be inherent in the units or items that are being measured, and some of the variability will result from the measurement system that is used.
- The measurement system will consist (minimally) of an instrument or gauge, and it often has other components, such as the operator(s) that uses it and the conditions or different points in time under which the instrument is used.
- There may also be other factors that impact measurement system performance, such as setup or calibration activities.
- The purpose of most measurement systems capability studies is to:
  - 1. Determine how much of the total observed variability is due to the gauge or instrument
  - 2. Isolate the components of variability in the measurement system 3
  - 3. Assess whether the instrument or gauge is capable (that is, is it suitable for the intended application)

R & R

- In this section we will introduce the two R's of measurement systems capability:
  - Repeatability (do we get the same observed value if we measure the same unit several times under identical conditions), and
  - Reproducibility (how much difference in observed values do we experience when units are measured under different conditions, such as different operators, time periods, and so forth).
- Other important aspects of measurement systems capability
  - The linearity of a measurement system reflects the differences in observed accuracy and/or precision experienced over the range of measurements made by the system.
  - Stability, or different levels of variability in different operating regimes, can result from warm-up effects, environmental factors, inconsistent operator performance, and inadequate standard operating procedure.
  - Bias reflects the difference between observed measurements and a "true" value obtained from a master or gold standard, or from a different measurement technique known to produce accurate values.

R & R

To introduce some of the basic ideas of measurement systems analysis (MSA) consider a simple but reasonable model for measurement system capability studies

$$y = x + \varepsilon \tag{8.23}$$

where y is the total observed measurement, x is the true value of the measurement on a unit of product, and ε is the measurement error. We will assume that x and ε are normally and independently distributed random variables with means μ and 0 and variances (σ_P^2) and (σ_{Gauge}^2), respectively. The variance of the total observed measurement, y, is then

$$\sigma_{\text{Total}}^2 = \sigma_P^2 + \sigma_{\text{Gauge}}^2 \tag{8.24}$$

Control charts and other statistical methods can be used to separate these components of variance, as well as to give an assessment of gauge capability.

An example

An instrument is to be used as part of a proposed SPC implementation. The quality-improvement team involved in designing the SPC system would like to get an assessment of gauge capability. Twenty units of the product are obtained, and the process operator who will actually take the measurements for the control chart uses the instrument to measure each unit of product twice. The data are shown in Table 8.6.

Figure 8.14 shows the $\bar{x}$ and R charts for these data. Note that the $\bar{x}$ chart exhibits many out-of-control points. This is to be expected, because in this situation the $\bar{x}$ chart has an interpretation that is somewhat different from the usual interpretation. The $\bar{x}$ chart in this example shows the **discriminating power** of the instrument—literally, the ability of the gauge to distinguish between units of product. The R chart directly shows the magnitude of measurement error, or the gauge capability. The R values represent the difference between measurements made on the same unit using the same instrument. In this example, the R chart is in control. This indicates that the operator is having no difficulty in making consistent measurements. Out-of-control points on the R chart could indicate that the operator is having difficulty using the instrument.

The standard deviation of measurement error, σ_{Gauge}, can be estimated as follows:

$$\hat{\sigma}_{\text{Gauge}} = \frac{\bar{R}}{d_2} = \frac{1.0}{1.128} = 0.887$$

The distribution of measurement error is usually well approximated by the normal. Thus, $\hat{\sigma}_{\text{Guage}}$ is a good estimate of gauge capability.

(*continued*)

An example (continued)

TABLE 8.6
Parts Measurement Data

Part Number	Measurements 1	2	$\bar{x}$	R
1	21	20	20.5	1
2	24	23	23.5	1
3	20	21	20.5	1
4	27	27	27.0	0
5	19	18	18.5	1
6	23	21	22.0	2
7	22	21	21.5	1
8	19	17	18.0	2
9	24	23	23.5	1
10	25	23	24.0	2
11	21	20	20.5	1
12	18	19	18.5	1
13	23	25	24.0	2
14	24	24	24.0	0
15	29	30	29.5	1
16	26	26	26.0	0
17	20	20	20.0	0
18	19	21	20.0	2
19	25	26	25.5	1
20	19	19	19.0	0
			$\bar{\bar{x}} = 22.3$	$\bar{R} = 1.0$

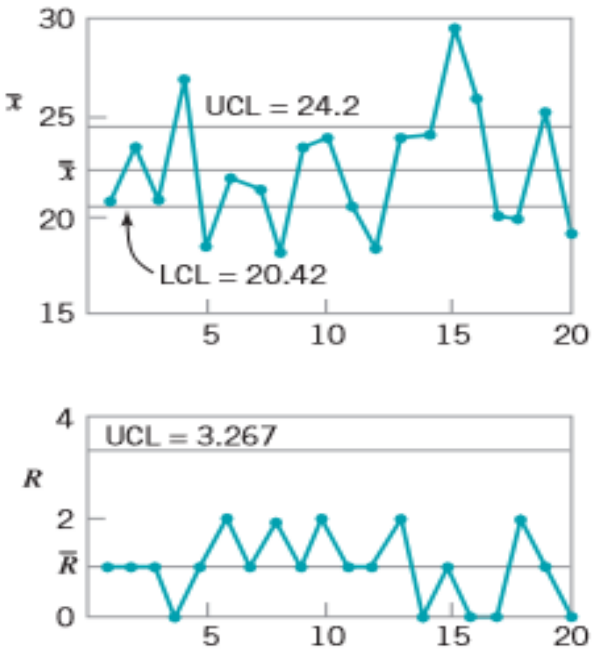


FIGURE 8.14 Control charts for the gauge capability analysis in Example 8.7.

Precision to Tolerance Ratio

- Precision to Tolerance Ratio

$$P/T = \frac{k\hat{\sigma}_{\text{Guage}}}{\text{USL} - \text{LSL}}$$

The part used in Example 8.7 has USL = 60 and LSL = 5. Therefore, taking $k = 6$ in equation (8.25), an estimate of the P/T ratio is

$$P/T = \frac{6(0.887)}{60 - 5} = \frac{5.32}{55} = 0.097$$

- Values of the estimated ratio P/T of 0.1 or less often are taken to imply adequate gauge capability.

Estimating variance components

We can use the data from the gauge capability experiment in Example 8.7 to estimate the variance components in equation (8.24) associated with total observed variability. From the actual sample measurements in Table 8.6, we can calculate $s = 3.17$. This is an estimate of the standard deviation of **total variability,** including both **product variability** and **gauge variability.** Therefore,

$$\hat{\sigma}^2_{\text{Total}} = s^2 = (3.17)^2 = 10.05$$

Since from equation (8.24) we have

$$\sigma^2_{\text{Total}} = \sigma^2_P + \sigma^2_{\text{Gauge}}$$

and because we have an estimate of $\hat{\sigma}^2_{\text{Gauge}} = (0.887)^2 = 0.79$, we can obtain an estimate of σ^2_P as

$$\hat{\sigma}^2_P = \hat{\sigma}^2_{\text{Total}} - \hat{\sigma}^2_{\text{Gauge}} = 10.05 - 0.79 = 9.26$$

Therefore, an estimate of the standard deviation of the product characteristic is

$$\hat{\sigma}_p = \sqrt{9.26} = 3.04$$

Accuracy and precision

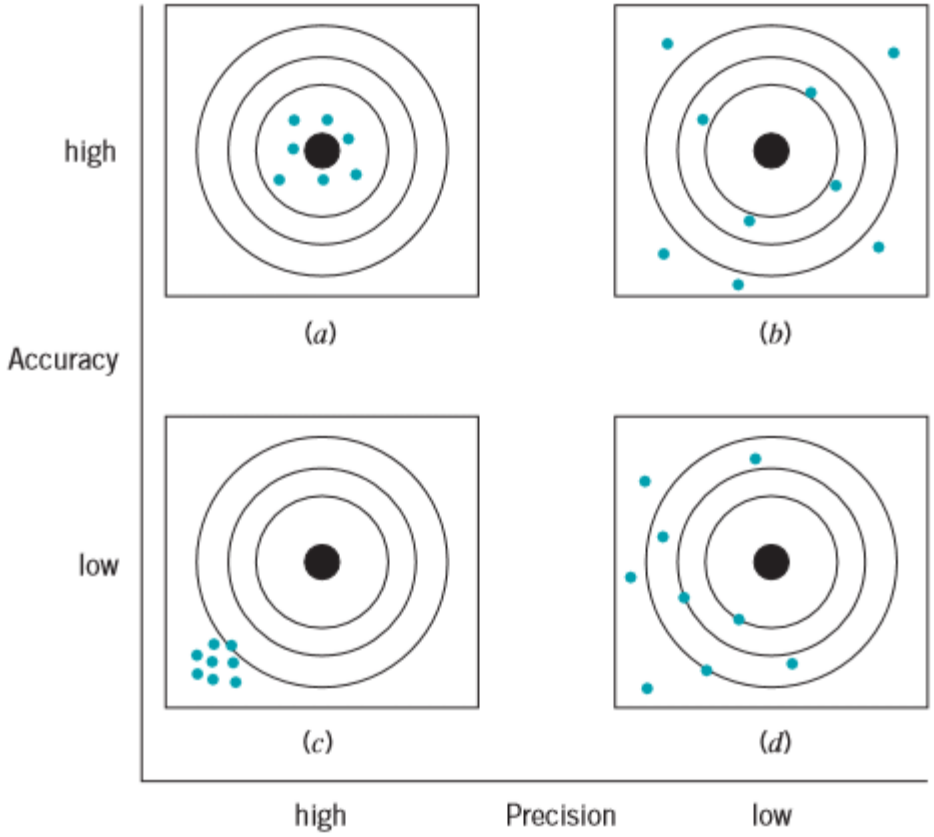


FIGURE 8.15 The concepts of accuracy and precision. (*a*) The gauge is accurate and precise. (*b*) The gauge is accurate but not precise. (*c*) The gauge is not accurate but it is precise. (*d*) The gauge is neither accurate nor precise.